TECHNICAL NOTE

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A Simplified Probability Equation for Gunshot Primer Residue (GSR) Detection

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ABSTRACT: The probability of finding gunshot primer residue particles when only a portion of a specimen is to be searched is calculated exactly, without the use of a simplifying assumption made by previous authors. Probabilities obtained by the exact method are compared with those published previously.

KEYWORDS: forensic science, gunshot residues, probability

One of the methods used for the detection of gunshot primer residue (GSR) is scanning electron microscopy with X-ray microanalysis. Particles are collected on a sticky lift (hereafter referred to as a specimen), which is searched for particles whose morphology and elemental composition are characteristic of GSR.

The search for GSR particles can be tedious and time-consuming. It is therefore of interest to know the probability of finding GSR particles on a specimen if a specified proportion of the specimen has already been searched without any GSR particles being detected.

Define the random variable **X** as the number of GSR particles found after a proportion **p** of a specimen has been searched. The desired probabilities can then be expressed as

$$P(\mathbf{X} = 0|\mathbf{p},\mathbf{N}) = \text{probability that no GSR particles would be found after searching a proportion } \mathbf{p} \text{ of a specimen, given that there are } \mathbf{N} \text{ GSR particles on the specimen}$$

This problem was considered by Wolten et al. [*1*]. In their study, the specimen was divided into 131 equal columns and it was assumed that GSR particles would be randomly distributed on the specimen. An additional simplifying assumption made was that if there were **N** GSR particles on the specimen, and $\mathbf{N} \leq 25$, then **N** columns would contain exactly one GSR particle and $131 - \mathbf{N}$ columns would be empty. Hence, using the

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simplifying assumption, and given **N** particles on the specimen and **c** columns searched, the desired probabilities become

$$P(\mathbf{X} = 0|\mathbf{p} = (\mathbf{c}/131),\mathbf{N}) = \frac{(131 - \mathbf{N})\,(131 - \mathbf{N} - 1)\,\ldots\,(131 - \mathbf{N} - \mathbf{c} + 1)}{131 \times 130 \times \ldots \times (131 - \mathbf{c} + 1)}$$

However, the additional simplifying assumption is not only unnecessary, it also breaks down quite rapidly. Given **N** GSR particles randomly distributed on a specimen, the probability that they occupy exactly **N** of the 131 columns is

$$\frac{131 \times 130 \times \ldots \times (131 - \mathbf{N} + 1)}{131^{\mathbf{N}}}$$

For **N** = 14, this probability is 0.487. Hence, if there are 14 or more GSR particles on the specimen, the probability that at least one column will contain more than one GSR particle is greater than 50%. For **N** = 25, the probability is over 90%.

The desired probabilities can be calculated without the simplifying assumption. It is only necessary to assume that the GSR particles are randomly distributed on the specimen.

Let there be **N** GSR particles on a specimen and assume proportion **p** of the specimen is to be searched. Define the random variable

$$\mathbf{X}_i = \begin{cases} 1 \text{ if the } i^{\text{th}} \text{ GSR particle is found} \\ 0 \text{ if otherwise} \end{cases}$$

for $i = 1, \ldots, \mathbf{N}$. Thus

$$P(\mathbf{X}_i = 1) = \mathbf{p}$$

$$P(\mathbf{X}_i = 0) = (1 - \mathbf{p})$$

for each $i = 1, \ldots, \mathbf{N}$, and each $\mathbf{X}_i$ is a binomial random variable with parameter **p**. Hence, if there are **N** randomly distributed GSR particles on a specimen, the probability of finding them in a search of a proportion **p** of the specimen can be regarded as a problem involving **N** independent, identically distributed binomial random variables with parameter **p**.

Define the random variable

$$\mathbf{X} = \sum_{i=1}^{\mathbf{N}} \mathbf{X}_i$$

The desired probabilities are then given exactly by

$$P(\mathbf{X} = 0|\mathbf{p},\mathbf{N}) = (1 - \mathbf{p})^{\mathbf{N}}$$

The values of $P(\mathbf{X} = 0|\mathbf{p},\mathbf{N})$ shown in Table 1 demonstrate the differences obtained when using the simplifying assumption described in the Aerospace Corp. report (by Wolten et al. [*1*]) and when using the exact solution derived above. Owens [*2*] described an error in the computer program used to generate the probabilities in the Aerospace report. The following tables therefore contain three columns: "Aerospace" contains the probabilities quoted in the Aerospace report, "Owens" contains the probabilities calculated by correctly applying the simplifying assumption made in the Aerospace report, and "Exact" contains the exact values.

TABLE 1—*Values of* $P(X = 0|p,N)$.

Percentage of Specimen Searched, p × 100	Aerospace	Owens	Exact
	N = 5		
0.76	0.96	0.96	0.96
7.6	0.67	0.67	0.67
15.3	0.30	0.43	0.44
22.9	0.083	0.27	0.27
30.5	0.013	0.16	0.16
38.2	0.001	0.086	0.090
	N = 10		
0.76	0.92	0.92	0.93
7.6	0.44	0.44	0.45
15.3	0.085	0.18	0.19
22.9	0.006	0.067	0.074
30.5	0.0001	0.022	0.026
38.2	1×10^{-6}	0.0065	0.0081
	N = 15		
0.76	0.89	0.89	0.89
7.6	0.28	0.28	0.31
15.3	0.02	0.071	0.083
22.9	0.0004	0.016	0.020
30.5	1×10^{-6}	0.0029	0.0043
38.2	6×10^{-10}	0.00043	0.00073
	N = 20		
0.76	0.85	0.85	0.86
7.6	0.18	0.18	0.21
15.3	0.006	0.027	0.036
22.9	2.3×10^{-5}	0.0034	0.0055
30.5	9×10^{-9}	0.00033	0.00069
38.2	3×10^{-13}	2.4×10^{-5}	6.6×10^{-5}
	N = 25		
0.76	0.81	0.81	0.83
7.6	0.11	0.11	0.14
15.3	0.001	0.0099	0.016
22.9	1×10^{-6}	0.00068	0.0015
30.5	5×10^{-11}	3.4×10^{-5}	0.00011
38.2	6×10^{-17}	1.1×10^{-6}	6.0×10^{-6}

Although the simplifying approximation breaks down rapidly, the probabilities calculated from it are, for all practical purposes, quite useful. However, the "Exact" method has these advantages:

1. Other than the random distribution of GSR particles on the specimen, no assumptions are made.

2. It is not necessary to divide the specimen into 131 (or any number of) columns. It is only necessary to specify the proportion of the specimen searched. At least one automated search program currently available allows the user to specify the percentage of the specimen area to be searched.

3. The required calculation can be performed in one step on any hand calculator having a y^x function.

References

[1] Wolten, G. M., Nesbitt, R. S., Calloway, A. R., Loper, G. L., and Jones, P. F., "Final Report on Particle Analysis for Gunshot Residue Detection," Report ATR-77(7915)-3, Aerospace Corp., Segundo, CA, 1977, pp. 93–98.

[2] Owens, A. D., "A Reevaluation of the Aerospace Corporation Final Report on Particle Analysis—When to Stop Searching for Gunshot Primer Residue (GSR)?" *Journal of Forensic Sciences*, Vol. 35, No. 3, May 1990, pp. 698–705.

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